Interactive Theorem Proving and Applications

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TP 4 - Inductive Constructs in Isabelle

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Exercice 1 (Inductive sets - Inductive Proofs)
Define the parameterized set of reacheable `path`'s inductively over a predicate `rel` stating that two points are related. The space of "points" is left abstract via parametric polymorphism,.

1. Either by the *specification construct* `inductive_set` or by `inductive` (predicate)
2. Instantiate `rel` with the successor relation on positive integer. Prove that there is a path from 11 to 13.
3. Prove que $3 \notin Even$
4. Prove transitivity for an arbitrary `rel` : if there is a path from `x` to `y`, and there is a path from `y` to `z`, there is a path from `x` yo `z`.

Objective : try first elementary Isabelle proof methods, so i.e. `subst`, `rule`, `rule_tac`, `erule`, `erule_tac` before applying more advanced methods like `simp` , `auto`, `metis`, and `sledgehammer` At the end, try to find the most compact version possible. You may experiment with Isar-style, declarative proofs.

Remark : A good balance between compactness and readability improves portability of your proof documents.

Exercice 2 (Modelling and Proof : The typed λ-calculus)
Define the λ-calculus as a data-type inside HOL. (This is also called a "deep embeding" into HOL). The first 3 parts are identical to TP 3.2.

1. Define the "terms" (abstract syntax tree) of the untyped λ-calcul as "data type"
2. Define the "types" (abstract syntax tree) du λ-calcul as "data type"
3. Define a function `instantiate` for that substitutes type-variables against types.
4. The environments Σ et Γ by using association lists.
5. Define inductively the well-typedness quartuple : a term t is well-typed with type τ in the environnements Σ et Γ.
6. Define a Σ_0 with the constants True, False, and equality inside our λ-calculus model.
7. Prove that in Σ_0 the encoding of the term $(\_ = \_)(True)$ has the (encoding of) the type $bool \to bool$.
8. Define Σ according to slide 30 in the module "U1 - λ-calculus" and prove that $(\_ = \_)(\_ = \_)$ is typeable in Σ.

Exercice 3 (OPTIONAL : Report)
(IN CASE THAT YOU WANT TO HAVE IT GRADED. RECALL THAT 2 OUT OF 6 TP's SHOULD BE SUBMITTED.)

1. Write a little report answering all questions above, note the difficulties you met, add some screenshots if appropriate. 3 pages max (except screenshots and other figures).